

Weak Scale in Heterotic String[†]

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Abstract

We investigate the possibility of lowering the string scale in four dimensional heterotic models possessing a non-perturbative extension of the gauge group. In particular, we consider a class of compactifications in which the perturbative gauge sector is massive, and all the gauge bosons are non-perturbative, with a coupling independent on the Planck and string scales.

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1. Introduction

A problem of string theory is the apparent huge separation between the natural string mass scale and the scale of weak interactions. In the $\mathcal{N}_4 = 4$ heterotic string, at the tree level the string scale is related to the Planck scale by the value of the gauge coupling, and a “normal” value of the latter implies that its value is close to the Planck mass. Unfortunately, this has the unappealing consequence that the string effects appear to be far from experimental detection. Scenarios with a low compactification scale [1] or a low (or intermediate) string scale have therefore been considered [2]–[8]. In the heterotic string, solutions to the problem of lowering the string scale such as taking into account loop corrections or the effect of the M-theory eleventh dimension [2] are characterized by the existence of bounds that prevent the string scale from being very (arbitrarily) small. In Ref. [8] it has been proposed that the problem of lowering the string scale, and having at the same time a value of the gauge coupling around $\sim 0,01$ or bigger, can be solved by assuming that the gauge group corresponding to the Standard Model belongs to a non-perturbative sector of the heterotic string. The existence of an enhancement of the heterotic gauge group, for a certain kind of compactifications, is a well known phenomenon: it emerged in the context of string-string duality, when considering the type I dual constructions [9, 10], in which this part of the gauge group appears perturbatively. On the heterotic side, the enhancement of the gauge group is explained by the existence of instantons that shrink to zero size [11]. The six dimensional coupling of this sector is therefore one, and in four dimensions the coupling depends on the volume of the further compactification, and not on the string and Planck scales. Indeed, as we observed in Ref. [12], in four dimensions the heterotic string possesses a further non-perturbative extension of the gauge group, due to the appearance, together with the small instantons of above, of gauge bosons deriving from six-dimensional massless tensors. The coupling of this sector does not depend on the volume of the compactification space, from six to four dimensions, but rather on its shape. This gauge sector is also a candidate to accommodate the Standard Model group.

The major problem related to this kind of scenarios is however the renormalization of the couplings, that in general leads to an effective gauge coupling depending on all the original bare ones, including that of the perturbative sector. This can eventually spoil the independence of the gauge coupling on the string scale.

In this note, we first discuss in some detail this problem. We then consider phases of heterotic compactifications for which this does not happen, being the gauge group *entirely* non-perturbative.

2. Discussion

In four-dimensional compactifications of the heterotic string, the tree level (perturbative) gauge coupling α_G is parametrized by the vacuum expectation value of the axion–dilaton field S :

$$\alpha_G^{-1} \sim \text{Im } S, \tag{2.1}$$

where $S = a + ie^{-2\phi}$. In terms of string parameters, this reads:

$$\alpha_G \sim \frac{V_{(6)}}{\lambda_H^{-2} l_H^6}, \quad (2.2)$$

where $V_{(6)}$ is the volume of the six-dimensional compact space \mathcal{M}_6 , l_H is the heterotic string scale and λ_H the heterotic coupling of ten dimensions. Owing to the relation:

$$G_N \equiv l_P^2 \sim \frac{\lambda_H^2 l_H^8}{V_{(6)}}, \quad (2.3)$$

there exist a tree level relation between the heterotic string scale, gauge coupling, and the Planck mass:

$$l_H = \langle \text{Im } S \rangle l_P = \alpha_{G(\text{tree})}^{-1} l_P. \quad (2.4)$$

According to this, the requirement of having a gauge coupling of order $10^{-2} \div 10^{-1}$ seems to imply that the heterotic string scale must be close to the Planck scale. As discussed in Ref. [8], the above statement is not anymore valid if the ‘‘Standard Model’’ gauge group is provided by small instantons, appearing in compactifications with reduced supersymmetry, such as for $\mathcal{M}_6 = T^2 \times K3$. The coupling $\alpha_{G'}$ of this sector depends on the volume of T^2 :

$$\alpha_{G'}^{-1} \sim \frac{V_{(2)}}{l_H^2}, \quad (2.5)$$

($V_{(6)} \equiv V_{(2)} V_{K3}$), and is independent on the string scale, still determined by (2.4). As we discussed in Ref. [12], T-duality of the heterotic string implies that, together with the above, there is another non-perturbative gauge group, with coupling given by the complex structure modulus of the torus:

$$\alpha_{G''}^{-1} \sim \frac{R_1}{R_2}. \quad (2.6)$$

This rather peculiar phenomenon tells us that, indeed, in the heterotic string there is a huge freedom in the choice of parameters, leading to a low string scale. However, generically one loop corrections to the perturbative gauge coupling modify the expression (2.1):

$$\frac{1}{\alpha_G} \approx \text{Im } S + \beta \Delta(T) + \sum_i \beta_i \Delta_i(U, Y) + f(T, U, Y), \quad (2.7)$$

where T and U are the moduli related respectively to the Kähler class and the complex structure of the two-torus ($\text{Im } T \sim V_{(2)} = R_1 R_2$, $\text{Im } U \sim R_1/R_2$), and β_i , $\beta = \sum_i \beta_i$ are beta-function coefficients depending on the specific gauge group factors, determined by the Wilson lines Y . Since the moduli T and U indeed parametrize the couplings of the two non-perturbative extensions of the gauge group, we interpret the dependence of α_G on these fields as due to the running of states charged under all these sectors. Based on this observation, we are led to conclude that an analogous phenomenon should happen also in the non-perturbative sectors, in which, generically, we expect that the gauge couplings given in Eqs. (2.5) and (2.6) indeed acquire a dependence on the modulus S . Therefore, even in case the Standard Model gauge group had a non-perturbative origin, there would

be a relation between the gauge coupling and string scale, leading, in the case of a non negligible dependence on the modulus S , to the loss of predictive power of the “tree level” considerations. The worse situation one can imagine is the one in which perturbative and non-perturbative sectors behave in an analogous way, with a simple exchange of the role of the fields S and T (or U). In this case, since $\Delta(T) \sim \text{Im } T$, $\Delta(U, Y) \sim \text{Im } U$ for large $\text{Im } T$, $\text{Im } U$, there would essentially be no difference between perturbative and non-perturbative sector. We notice that, among the very special situations in which this mixing of couplings does not occur, there is the case in which the gauge group is $U(16)$, namely the one for which heterotic/type I duality has been tested in Ref.[13]. The choice of such configurations seems however to put a too severe constraint on the allowed gauge groups.

It is however possible to escape this problem, by considering heterotic configurations in which the entire gauge group has a non-perturbative origin. An indication that such configurations exist is given by the expression of the correction to the coupling of the R^2 term in an heterotic $\mathcal{N}_4 = 2$ models without perturbative gauge group [14, 15], that read:

$$\frac{1}{g_{grav}^2} \sim \text{Im } S - \log \text{Im } T |\vartheta_2(T)|^4 - \log \text{Im } U |\vartheta_2(U)|^4, \quad (2.8)$$

where for simplicity we don’t specify normalization coefficients and the term accounting for the infrared running (see Refs. [15]–[17]). In the limit of large $\text{Im } T$, $\text{Im } U$, the second and third term behave linearly in $\text{Im } T$, $\text{Im } U$, signaling the appearance of new massless states in the corresponding non-perturbative sectors, of which these moduli parametrize the couplings. Indeed, there exists a type IIA/B self-mirror orbifold that could describe the dual of this phase: it was constructed in Ref. [18], as a semi-freely acting $Z_2 \times Z_2$ orbifold [19], corresponding to a singular limit in the moduli space of the compactification on the so-called Del Pezzo surface [20]. This CY^{19,19} manifold is a double fibration over \mathbf{P}^1 . At the Z_2 orbifold point, this model has two twisted sectors, each one providing eight vector multiplets and eight hyper multiplets. According to Ref. [18], the corrections to the R^2 term read:

$$\frac{1}{g_{grav}^2} \sim -\log \text{Im } T^1 |\vartheta_4(T^1)|^4 - \log \text{Im } T^2 |\eta(T^2)|^4 - \log \text{Im } T^3 |\eta(T^3)|^4, \quad (2.9)$$

where T^1, T^2, T^3 are the moduli associated to the Kähler classes of the three tori into which the compact space is divided. It is natural to identify the moduli T^1, T^2, T^3 respectively with the heterotic fields S, T and U in the phase in which non-perturbative massless sectors appear. The theta function in the first term indicates in fact that the corresponding sector, the perturbative sector of the heterotic string, is massive, with a mass scaling roughly as $\sim \text{Im } S$. For large $\text{Im } S$, the contribution of the first term diverges logarithmically, indicating, as usual [21], the “disappearance” of the corresponding sector, or, in other terms, the fact that its states are infinitely massive. We expect that this behavior reflects in the gauge couplings of the non-perturbative sectors. More precisely, since in this $\mathcal{N} = 2$ model the gauge group is realized at the level 2, with an equal number of vector and hyper multiplets, and the type II compactification manifold is self-mirror, we don’t expect corrections to the moduli spaces associated to these states. There should be no corrections to the gauge couplings either, that should be given by their “bare” value, as a function of the only field

T or U respectively. However, even in more realistic situations, such as those in which supersymmetry is further broken to $\mathcal{N} = 1$ or $\mathcal{N} = 0$, there should not be strong corrections to the gauge couplings depending on the field S , because the states of this sector are infinitely massive. In this class of theories, therefore, we expect the strength of the gauge coupling to be independent on the string scale, that can be arbitrarily low.

In the opposite limit $\text{Im } S \rightarrow 0$, namely in the S-dual situation, a better description is given in terms of $\tilde{S} \equiv -1/S$. The first term in Eq. (2.9) changes according to $\vartheta_4(S) \rightarrow \vartheta_2(\tilde{S})$, and in the large $\text{Im } \tilde{S}$ limit the first term diverges linearly in $\text{Im } \tilde{S}$, indicating that the states of the corresponding sector, the perturbative sector of the S-dual heterotic theory, are close to become massless ($m \sim 1/\tilde{S}$). As discussed in Ref. [22], this model is in fact probably connected to the ordinary $\mathcal{N} = 2$ heterotic orbifold with a rank 16 perturbative gauge group, and two equivalent non-perturbative sectors (cfr. Ref. [12]). This model is dual to a $Z_2 \times Z_2$ non-freely acting type II orbifold. The connection, at the limit $\text{Im } \tilde{S} \rightarrow \infty$, should however involve some kind of phase transition, in which not only states of the “ S ” sector become massless, but also in the two non-perturbative sectors new gauge bosons appear, extending the rank of the gauge group from 8 to 16 in each of these sectors.

3. Conclusions

In this paper we considered the problem of lowering the string scale in the heterotic string, without lowering the gauge coupling, that should remain of the order $\sim 0,01$. We found that indeed this is possible in a wide class of configurations. This is made possible by the fact that heterotic compactifications with reduced supersymmetry possess several non-perturbative extensions of the gauge group, for which the gauge coupling escapes the usual tree level relation to the Planck and string scales. An interesting class of compactifications is the one in which the entire gauge group is non-perturbative. Indeed, using type II/heterotic duality, it is possible to see that, in the heterotic moduli space, perturbative and non-perturbative sectors are essentially equivalent [12]. There is therefore no reason for preferring one sector among the others to be the one giving origin to the Standard Model gauge group. The latter could well lie on a non-perturbative sector of the heterotic string, as proposed in Ref. [8].

Our analysis was limited to $\mathcal{N}_4 = 2$ compactifications. When supersymmetry is further broken to $\mathcal{N}_4 = 1$, we expect on general grounds new non-perturbative sectors to appear, parallel to the appearance of new moduli entering in the expressions of string threshold corrections, that we can associate to the couplings of these additional sectors. This makes the scenario even richer. On the other hand, non-perturbative phenomena have been shown to play a crucial role in $\mathcal{N}_4 = 1$ heterotic compactifications, in which most probably they are responsible for a further, complete breaking of supersymmetry [23].

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